Granny's Balloon Trip

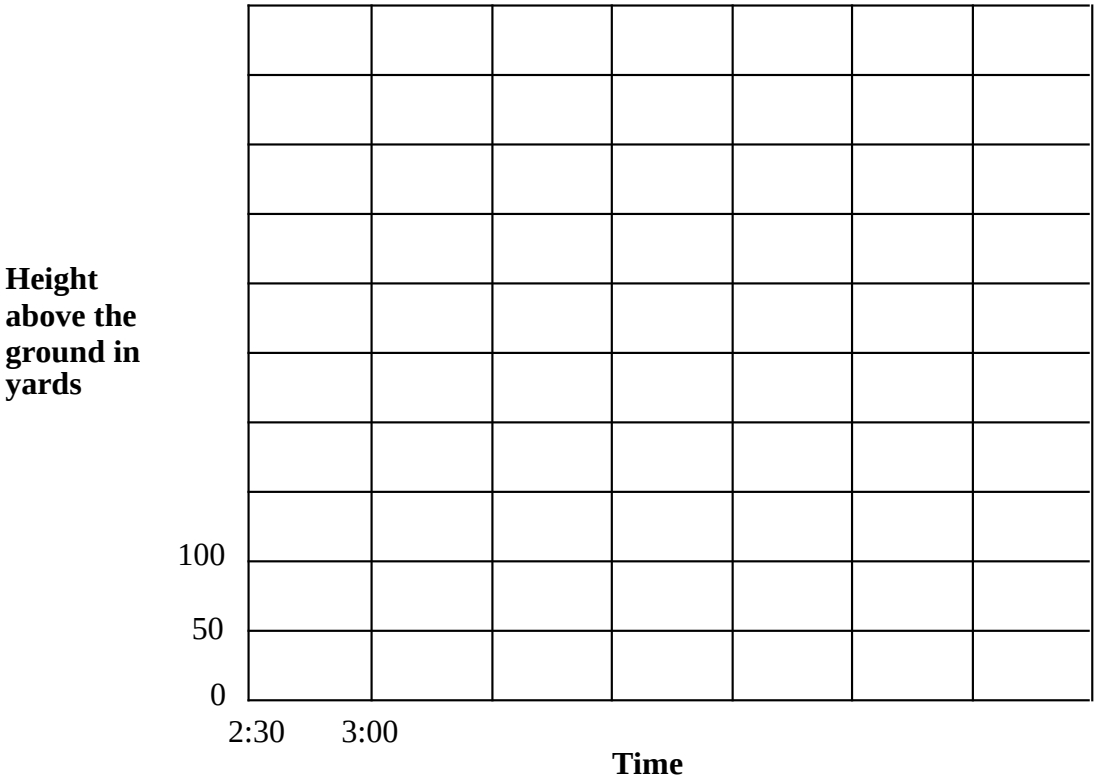
This problem gives you the chance to:
- represent data using tables and graphs

On her eightieth birthday, Sarah's granny went for a trip in a hot air balloon.

This table shows the schedule of the trip.

Time	2:30	3:00	3:30	4:00	4:30	5:00	5:30	6:00
Height above the ground in yards	0	150	250	350	500	250	100	0

1. Finish labeling the axes and draw a line graph to show the balloon trip.



2. For about how long did the balloon stay above 250 yards?

_____

3. At about what time do you think the balloon rose to 400 yards?

_____

Explain how you figured this out.

_____

_____

_____

4. At about what height do you think the balloon was at 5:50?

_____

Explain how you figured this out.

_____

_____

_____

Task 4: Granny's Balloon Trip	Rubric	
The core elements of performance required by this task are: • represent data using tables and graphs Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Completes both axes correctly Completes the line graph correctly *Partial credit*. One point for a partly correct graph such as: Correct points marked but line not drawn. or Line drawn but one or two incorrect points marked.	1 2ft (1)	3
2. Gives correct answer: **One and a half hours**	1ft	1
3. Gives correct answer: accept 4:05 through 4:15 Gives a correct explanation such as: If you follow the line for 400 yards on the vertical axis and look down the horizontal axis it is about half way between 4:00. and 4:30.	1ft 1ft	2
4. Gives correct answer: accept 5 through 35 yards above the ground Gives a correct explanation such as: It was less than 50 yards and more than 0 yards	1ft 1ft	2
Total Points		**8**

Granny's Balloon Trip

Work the task. Look at the rubric. What are the big mathematical ideas of the task?

How often do students in your class get opportunities to make their own graphs versus reading information off the graphs provided?
How is the thinking needed to construct a graph different that the thinking needed to read or interpret a graph?
What opportunities have your students had to work with line graphs?

- In the development of the idea did they discuss when one type of graph would be more appropriate than the other?
- Why are items on the horizontal axis of a line graph located on the grid lines and items on the horizontal axis of a bar graph located between grid marks? Do you think your students understand this distinction?

Do the graphs used in class give students opportunities to estimate values that don't fall evenly on the grid lines?

Look at the scales made on the horizontal axis. How many of your students:

- Used increasing intervals of 50?
- Used increasing numbers with different size intervals?
- Numbers on the scale matched data points rather than measuring out intervals
- Included increasing and decreasing numbers?
- Repeated some values

How many of your students made bar graphs instead of line graphs? Why don't these make sense for this situation?
How many of your students did not fill in the points after they made the labels on the axes?
How many of your students didn't fill in the lines? Did this effect their answers on other parts of the task or were they able to get enough information from the table?

Now look at student work on part 2, finding the length of time the balloon was above 250 yds. How many of your students put:

1 1/2 hr.	30 min.	1 hr.	3:30	2 hr.	2:30	5:00	Other

What might a student be thinking to get those answers? What kind of question could you pose to the class to get them to see the errors in their logic?

To answer the final question, students needed to think about dividing the time between 5:30 and 6:00 into 10 min. intervals to locate 5:50 and then read across and estimate distance on the vertical axis. Look at your student work on part 4. How many of your students put:

Correct estimate	Number larger than 50	Number larger than 100	Exactly 50	0	Time notation e.g. 4:45 or 5:30	Other

What do students need to understand about time and about spatial visualization in order to solve this task?

Can you look at some of the student work to see what misconceptions may have led them to their answers? Usually there is a logic to incorrect errors. Understanding this logic can help us plan out the types of experiences that will help students let go of their thinking and adopt better strategies.

Looking at Student Work on Granny's Balloon

Student A is able to label the axes in appropriate, equal size intervals matching the scale already being used. The student shows visually how he used estimation to answer questions 3 and 4. Notice that the student labeled ten-minute intervals to find the 5:50 and then work backwards to find the correct number of yards.

Student A

This table shows the schedule of the trip.

Time	2:30	3:00	3:30	4:00	4:30	5:00	5:30	6:00
Height above the ground in yards	0	150	250	350	500	250	100	0

1. Finish labeling the axes and draw a line graph to show the balloon trip.

Student A, part 2

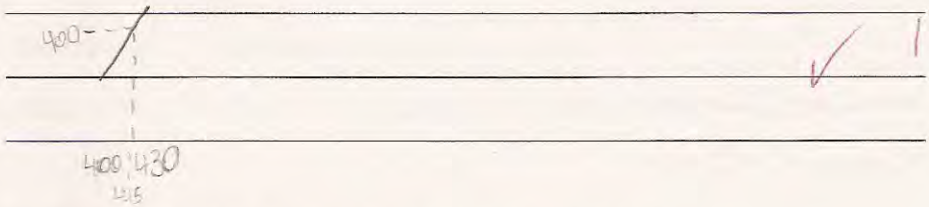
2. For about how long did the balloon stay above 250 yards?

1:30 ✓ 1

3. At about what time do you think the balloon rose to 400 yards?

4:15 ✓ 1

Explain how you figured this out.

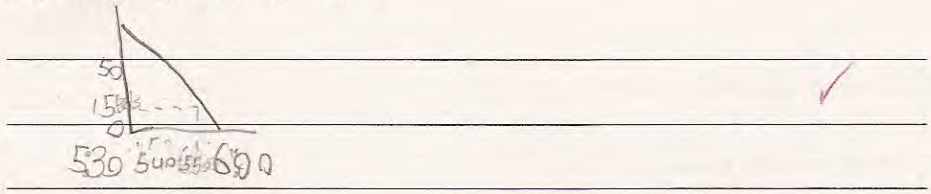


✓ 1

4. At about what height do you think the balloon was at 5:50?

250yds ✓ 1

Explain how you figured this out.



✓ 1

Student B has 7 out of the 8 points on the task. The student has difficulty thinking about the time the balloon is above 250 yds. The student only considers the time the balloon is still rising, forgetting that part of the descent is also above 250 yd. *What question could you pose to the class to push their thinking about this issue? What other questions could you ask to have students think about a range of values on the graph?*

Student B

2. For about how long did the balloon stay above 250 yards?

The balloon ~~got to~~ reached 500 yd.s by 4:30. It reached 250 at 3:30. It was above 250 yd.s for 1 hour. = 1 hour x 0

Estimating values between grid marks was challenging for many students. Student C has a reasonable strategy for estimating the distance in part 3 (althoughthe ten minutes should have been added to 4:00). However this same strategy does not work for finding the value in part 4. *Can you explain why it no longer works?*

Student C

3. At about what time do you think the balloon rose to 400 yards?

10 min. ✓ 1

Explain how you figured this out.

I figured this out by breaking up how long it took to get from 350 ft to 500 ft. and it took 30 min so I Broke that down and got 10 min. ✓ 1

4. At about what height do you think the balloon was at 5:50?

50 ft. X 0

Explain how you figured this out.

I did the same thing as what I did above except for the part where that is adding and this is subtracting. x 0

Why is dividing by 2 incorrect for finding the distance for 5:50 Is there way to divide 100 to get a goodestimate? Lookat the work of StudentD.

3. At about what time do you think the balloon rose to 400 yards?

about 4:15 ✓ 1

Explain how you figured this out.

At 4:00 it was at 350 yards above the ground. At ✓ 1
4:30 it was at 500 yards. So 4:15 is right in the middle of those 2.

4. At about what height do you think the balloon was at 5:50?

50 feet X 0

Explain how you figured this out.

At 5:30 the ballon was at 100 feet going down. X 0
At 6:00 it was at 0 feet above the ground.
50 feet is right in the middle of those 2.

While Student E's estimate for part 3 is within the allowable range, what is the student not understanding about proportional thinking?

Student E

3. At about what time do you think the balloon rose to 400 yards?

About 4:15 ✓1

Explain how you figured this out.

I guessed because I thought it would be in the of 4:00 and 4:30 because ✓1 400 was in the middle of 350 and 500.

4. At about what height do you think the balloon was at 5:50?

50 yards +x 0 0

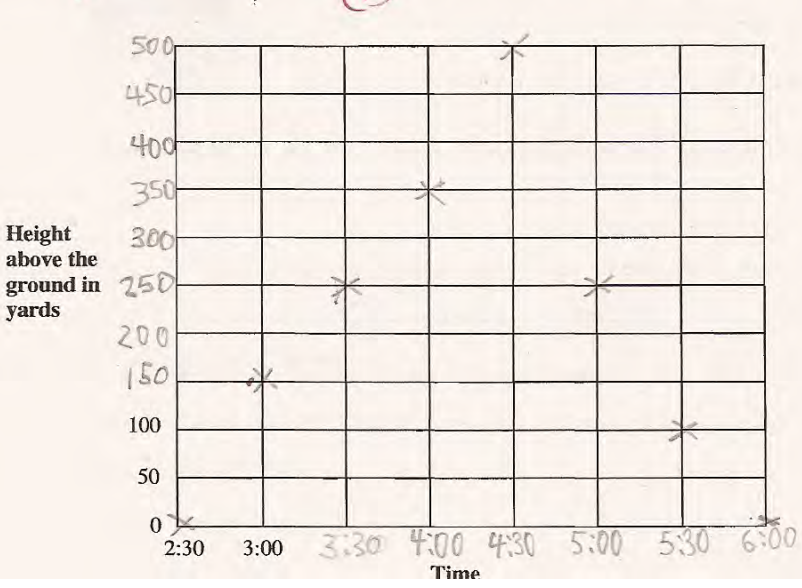
Explain how you figured this out.

50 is in the middle of 100 and 0 and so is 5:50. +x 0 0

Student F is able to label the axes in equal intervals and graph the data points. The student does not show the line for a line graph, which may have made it more difficult to estimate values for other parts of the graph.

Student F

1. Finish labeling the axes and draw a line graph to show the balloon trip.



Student F, part 2

2. For about how long did the balloon stay above 250 yards?

30 min. ✗ 0

3. At about what time do you think the balloon rose to 400 yards?

4:15 ✓ 1

Explain how you figured this out.

I went in the middle of 4:00 and 4:30. ✗ 0

4. At about what height do you think the balloon was at 5:50?

30 yards ✓ 1

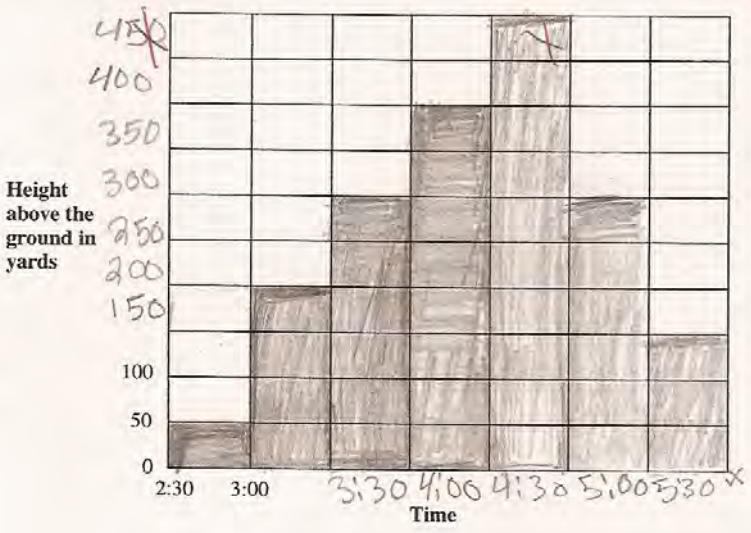
Explain how you figured this out.

I subtracted 70 from 100. ✗ 0

Student G makes a bar graph instead of line graph. The student seems to want the vertical scale to be between lines, which would imply that there aren't values in between (the whole row would be 150 or 300). The same is true for the horizontal scale. Something is either at 3:30 or 4:00, but there are no values in between. It's not possible with this graph to estimate a height for the balloon at 3:45.

Student G

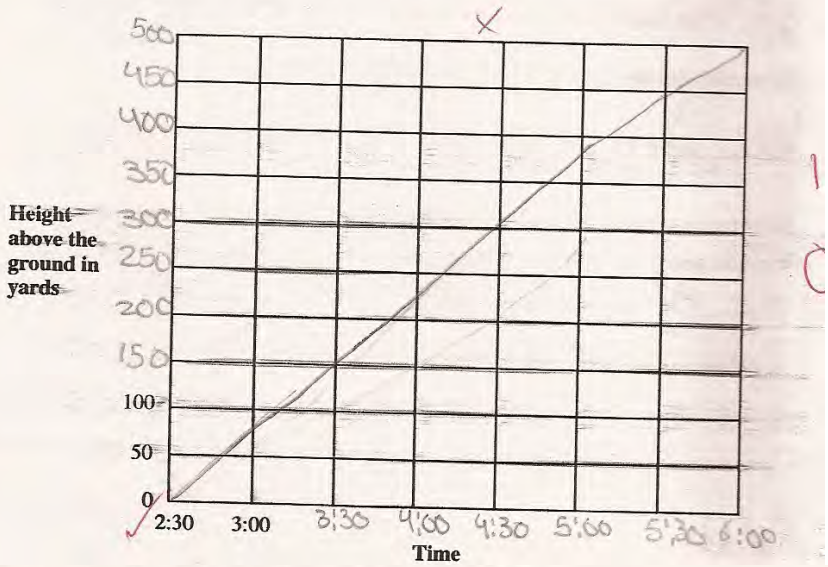
1. Finish labeling the axes and draw a line graph to show the balloon trip.



Some students think all line graphs are linear. Student H scales the vertical axes in equal intervals, but doesn't use the data in the table to plot the graph. The student does seem to look at the line in attempting to answer parts 3 and 4. *Why do you think a student might have this type of misconception about graphing? What questions might you pose to the class?*

Student H

1. Finish labeling the axes and draw a line graph to show the balloon trip.



2. For about how long did the balloon stay above 250 yards?

3:00 X O

3. At about what time do you think the balloon rose to 400 yards?

5:00 X O

Explain how you figured this out.

I figured this out by look at my chart in the backe page. X O

4. At about what height do you think the balloon was at 5:50?

X O

450 and 500

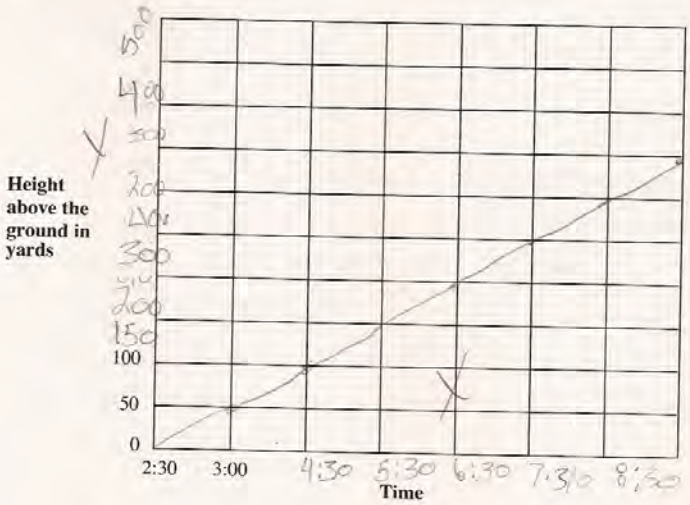
Explain how you figured this out.

Because at 6:00 and 6:50 it 500 and almost 550 X O

Student I also makes a linear graph. However this student does not divide the horizontal scale into equal intervals or even ascending values. Notice the student skips from 3:00 to 4:30 on the horizontal axis. Both students just end the graph at the highest value 500. In contrast to Student H, Student J does not seem to use either the table or the graph in answering questions about the trip. *What kinds of experiences does this student need? What might be logical next steps?*

Student I

1. Finish labeling the axes and draw a line graph to show the balloon trip.



2. For about how long did the balloon stay above 250 yards? 5 1/2

3. At about what time do you think the balloon rose to 400 yards? 20/30

Explain how you figured this out.

I calculated in my mind and this is how I did it.

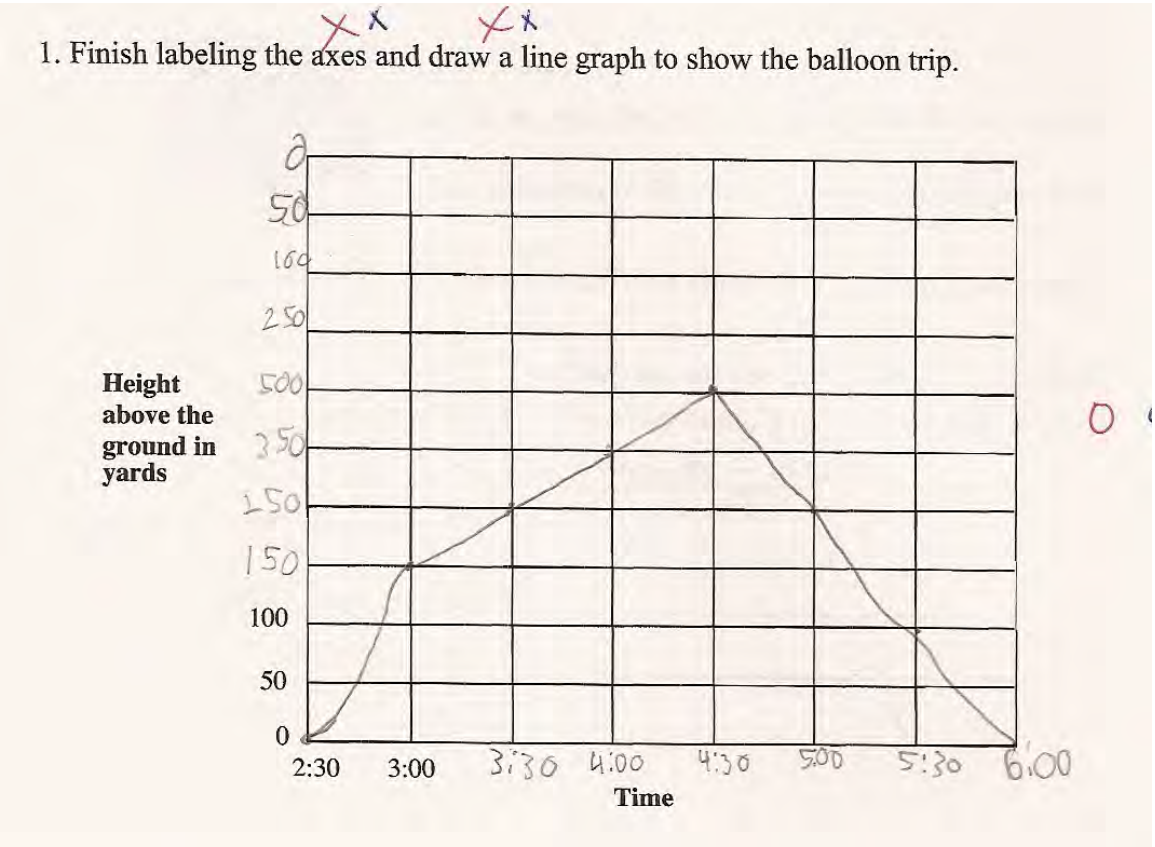
4. At about what height do you think the balloon was at 5:50? 5/50

Explain how you figured this out.

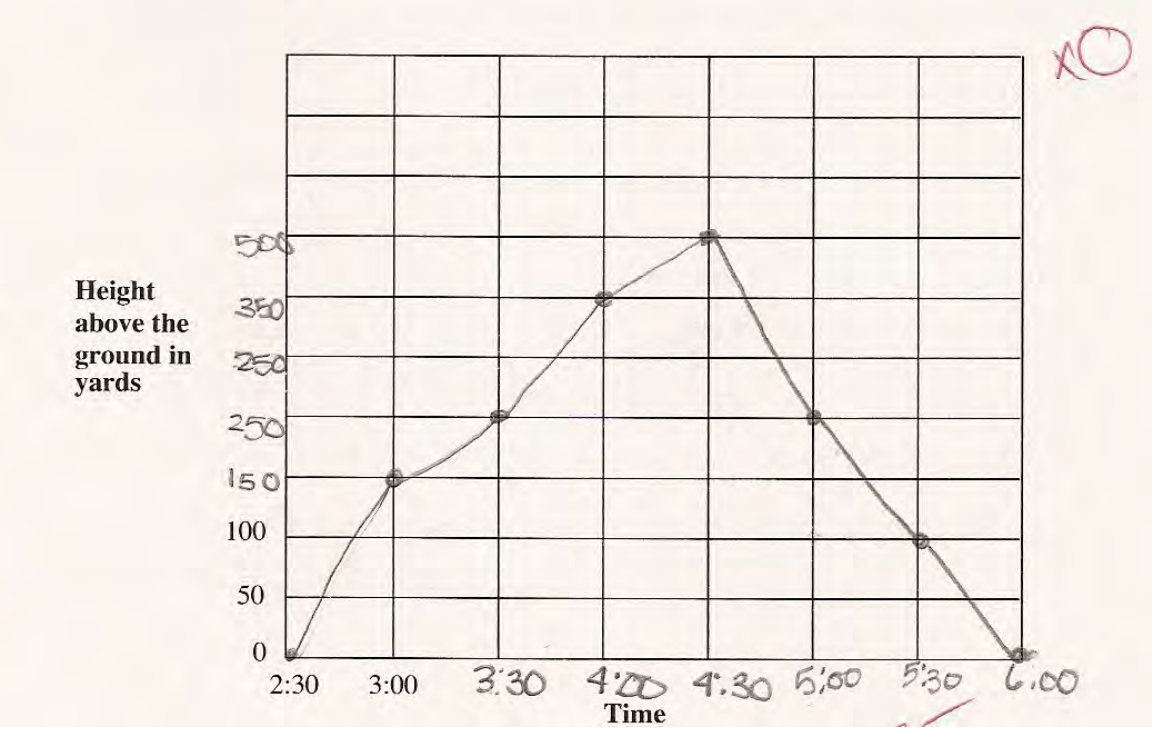
I did it the same way that I did the other one

Understanding how to scale the axes was quite challenging for many students. While Student K has values at unequal intervals on the vertical axis and numbers that return to 0, the student ignores the top of the scale to at least graph the points appropriately for the incorrect scale. Student L has unequal intervals and 250 is listed twice. Student M has listed the data values instead of providing intervals on the vertical axis. Student N has 2 zeroes on the vertical axis and skips from 3:00 to 5:30 on the horizontal axis. The line on the graph does not match the action of the data table.

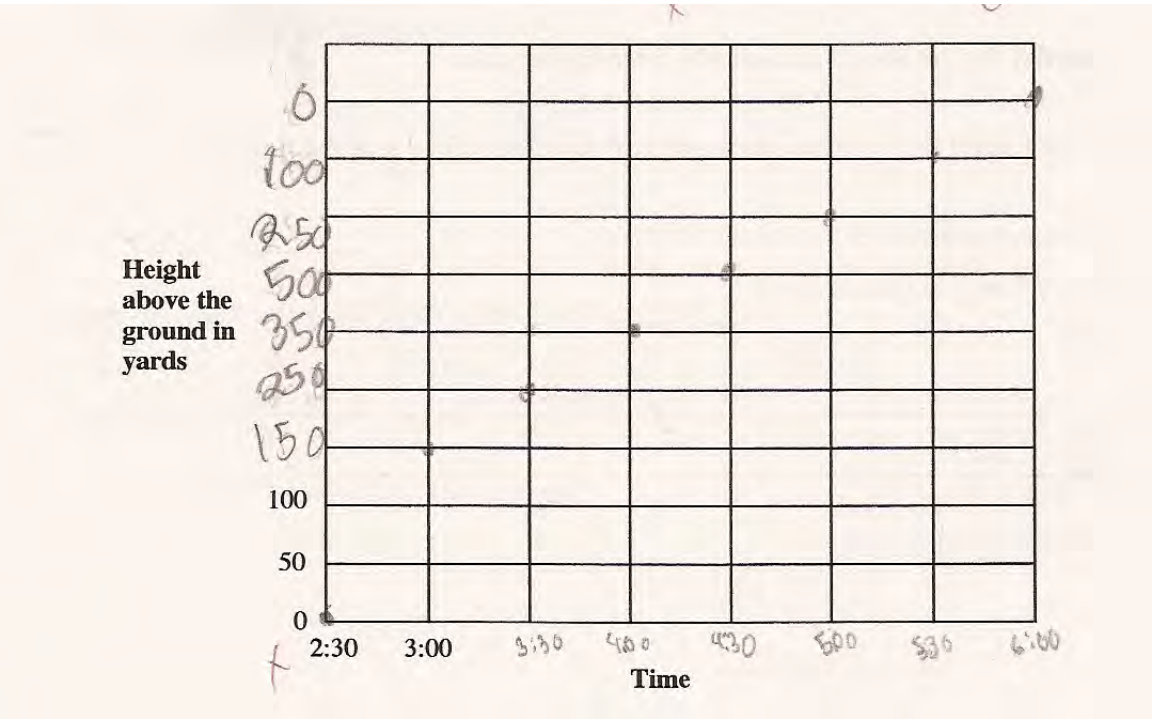
Student K



Student L

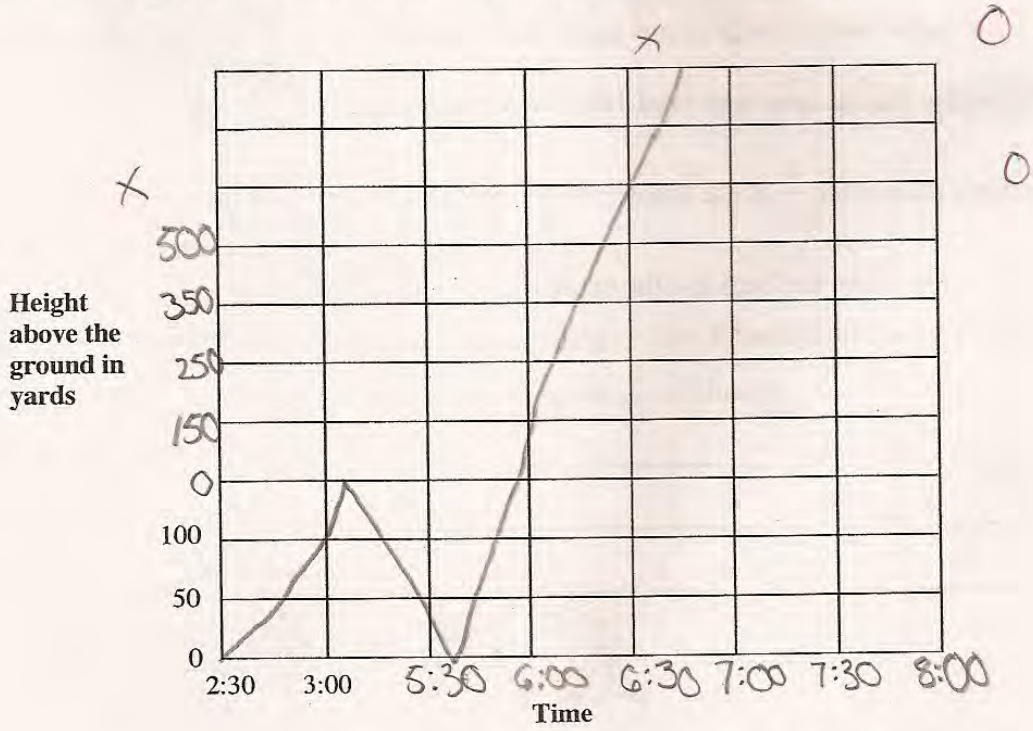


Student M



Student N

1. Finish labeling the axes and draw a line graph to show the balloon trip.



5th Grade Task 4 Granny's Balloon Trip

Student Task	Represent data using tables and graphs. Interpret information on a line graph and estimate values between grid lines.
Core Idea 5 Data Analysis	**Display, analyze, compare and interpret different data sets.** • Organize and display data in appropriate graphs and representations (e.g., histograms and line graphs).

Based on teacher observations, this is what fifth graders knew and were able to do:

- Label the axes
- Estimate the time when the balloon reached 400 ft.
- Graph points on a graph from a data table

Areas of difficulty for fifth graders:

- Making a scale with equal intervals
- Determining when to use a point, line, or bar graph.
- Scaling between intervals to find the distance at 5:50.
- Thinking about a range of values, finding how long the balloon was above 250 yards.

MARS Test Task 4 Frequency Distribution and Bar Graph, Grade 5

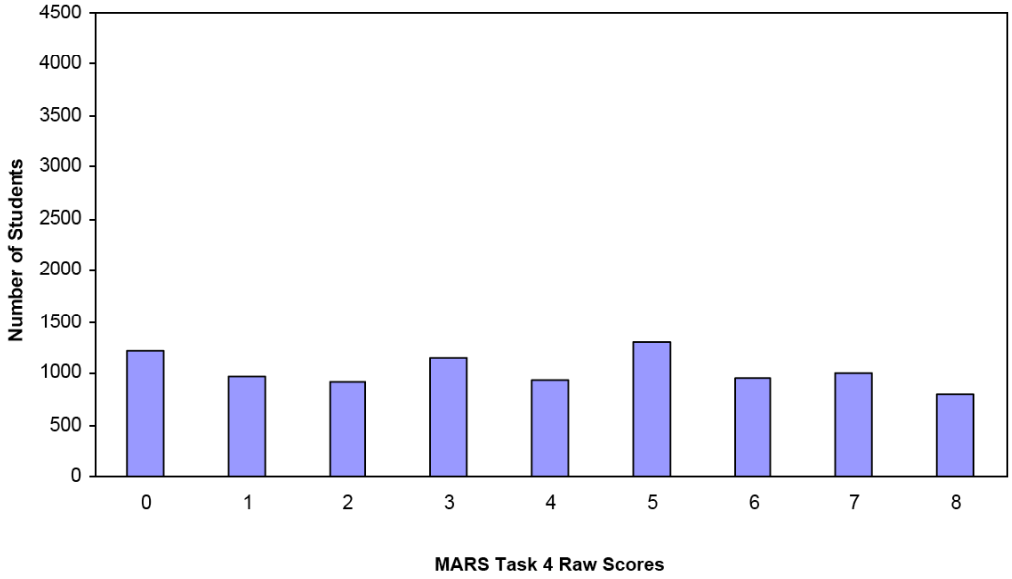
Task 4 - Granny's Balloon Trip

Mean: 3.85 StdDev: 2.54

Table 28: Frequency Distribution of MARS Test Task 4, Grade 5

Task 4 Scores	Student Count	% at or below	% at or above
0	1225	13.2%	100.0%
1	968	23.7%	86.8%
2	916	33.5%	76.3%
3	1156	46.0%	66.5%
4	940	56.2%	54.0%
5	1311	70.3%	43.8%
6	943	80.5%	29.7%
7	1006	91.3%	19.5%
8	802	100.0%	8.7%

Figure 37: Bar Graph of MARS Test Task 4 Raw Scores, Grade 5



The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.
Most students, 76%, were able to label the axes in equal intervals (students with a score of 1 or 3 were able to do this, but not students with a score of 2). Many students, 76%, could estimate a reasonable time for the balloon reaching a height of 400 yards by using their graph or the table. More than half the students, 54%, could label the axes, graph most of the points, and make an estimate about time for reaching 400 yd. Some students, 20%, could make the graph and estimate the time to reach 400 yd. and the height of the balloon at 5:50. Almost 9% of the students could meet all the demands of the task, including nding how long in total the balloon was above 250 yd. 13% of the students scored no points on the task. 83% of the students with this score attempted the task.

Granny's Balloon Trip

Points	Understandings	Misunderstandings
0	83% of the students with this score attempted the task.	Students did not understand how to label the axes. 21% had unequal intervals such as going by 50's, 100's, and 200's. 10% had numbers on scale out of sequence. 10% repeated numbers, such as two 250's or two 0's.
1	Students could label the axes.	Students had trouble graphing the points. Almost 8% did not draw lines. 8% had points o because of the strange scales. 8% put no points on the graph. 13% made bar graphs. About 5% of the students made a graph that was a straight line going up from 0 to 500. About 6% made a line graph that didn't match the action of the story.
2	Students could estimate the time when the balloon reached 400 yds. and explain how they gured it out. They may have reasoned from their graph or from the table.	About 4% of the students made each of the following incorrect answers: 4:00, 4:20, 4:25,4:30, and 4:50.
4	Students could label the axes, graph most of the data points, and estimate when the balloon would reach 400 yds.	Students had more di culty thinking about the height of the balloon at 5:50, which involved estimating on both scales. Dividing the interval into thirds was di cult. 21% just divided the nal line segment in half and got an answer of 50 yds, which is actually the distance at 5:30.
5	Students could label the axes, graph all the points accurately and draw line to connect the data points. They were able to estimate the time the balloon would reach 400 yds.	Students could not estimate the height at 5:50 or describe the time the balloon was above 250 yds.
7	Students could make the graph and estimate the time the balloon would reach 400 yds. and the height of the balloon at 5:50.	Students had di culty with the idea of above 250 yds. 17% thought it was only 30 min. 14% thought it was 1 hour. 5% thought it was 2 hr. Almost 8% put a clock time of 3:30 instead of the amount of elapsed time. 10% put other clock times.
8	Students understood the idea of scale as equal intervals and could label the axes, graph the data points and draw the line. They could estimate using their line graph and reason about elapsed time.	

Implications for Instruction

Students at this grade level should have many opportunities to make their own graphs. The thinking involved in planning the scale, labeling the axes, and setting up the graph is quite different from that required to just read information off a graph. Students at this grade level should be comfortable with a variety of graphs besides just bar graphs, including line graphs, pie graphs, and histograms. Students should start to be able to reason about which one to choose for different situations or why a particular type of graph would be inappropriate for the situation. They need to learn about attributes of graphs.

Students should also be able to think about scale in increments other than one; e.g. like 10, 50, 100, 30, etc. They should also realize the difference between the scale, marking off equal size groups, and the data points, the information graphed or plotted in the interior of the grid. Many students don't realize that the scale should increase in size. The action of increasing and decreasing takes place with the data points. Understanding that the scale is a framework and the plotted points tell the action of the information is difficult for students. Some students want to draw lines going back to the origin if something goes back to zero. There are so many things to consider when making a time and distance graph. For example, take some information about a walk to school or a bicycle trip. Have students graph the information as distance from home. Then take the same information and graph it as total distance traveled. Have them discuss how the same information can look so different. (See MARS task 2006- Sixth Grade- Bike Ride)

Ideas for Action Research

Making Graphs for Stories

Having students make their own graphs about situations helps them to understand the logic of the graph and see how the lines do not represent the action of the story. Consider giving your students some story situations and have them make a graph of the general situation, not necessarily dealing with the issues of an exact scale. For example:

> A factory cafeteria contains a vending machine selling drinks.
> On a typical day:
> - The machine starts half full.
> - No drinks are sold before 9 a.m. or after 5 p.m.
> - Drinks are sold at a slow rate throughout the day, except during the morning and lunch breads (10:30-11 am and 102 pm) when there is a greater demand.
> - The machine is filled up just before the lunch break. (It takes about 10 minutes to fill).
>
> Make a sketch to the graph to show how the number of drinks in the machine might vary from 8 am to 6 pm.

What does the student have to understand about graphing to do this task? What do you think the graph might look like? What errors do you anticipate students might make? How does this help you think about how to process this activity? What are the mathematics you want to bring out or highlight as students discuss their work?

Performance Assessment Task

Granny's Balloon Trip
Grade 5

This task challenges a student to use knowledge of scale to organize and represent data from a table on graph. A student must understand scale as being comprised of equal-sized units to label the axes on the graph. A student must understand what type of graph (point, line or bar) best suits the given data set to make the appropriate graph. A student must be able to estimate distances between the grid lines to plot points and interpret the data to answer questions. A student must interpret information on a line graph and estimate values between grid lines and think about range solve problems about the data.

Common Core State Standards Math - Content Standards

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given pint in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to gravel from the origin in the direction of one axis, and the second number indicates how far to travel from the origin in the direction of the second axis, with the convention that the names of the two axes and coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y- coordinate).

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

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Grade Level	Year	Total Points	Core Points	% At Standard
5	2007	8	4	54%